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Predictive Event-Triggered Control for Disturbanced Wireless Networked Control Systems*

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Abstract The control and scheduling for wireless networked control system with packet dropout and disturbance are investigated. A prediction based event triggered control is proposed to reduce data transmissions while preserving the robustness against external disturbance. First, a trigger threshold is especially designed to maintain the difference of the estimated and actual states below a proper boundary when system suffers from packet dropout. Then a predictive controller is designed to compensate for packet dropouts by utilizing the packet-based control approach. The sufficient conditions to ensure the closed-loop system being uniformly ultimately bounded are derived, with consequently the controller gain method. Numerical examples illustrate the effectiveness of the proposed approach.

Keywords Event-trigger, packet-based control, wireless networked control systems.

1 Introduction

Networked control systems (NCSs) have long been a research focus in the past two decades, thanks to their unique advantages such as the much ease of implementation, the high flexibility of system configuration, the low cost of maintenance, etc.[1–7]. In recent years, with the fast development of wireless communication technology as well as the embedded computing devices, we have witnessed a fasinating "wireless" trend of NCSs, where various wireless communication networks are used to replace the data channels in NCSs, forming a new generation of NCSs, i.e., wireless networked control systems (WNCSs)[8–11]. Various application examples of WNCSs have already been seen in smart building[12], internet of vehicles[13], Industrial 4.0[8], and so on, and it is a generally held belief that the development of the WNCSs theory can be vital in the next era of the information technology.

Not surprisingly, the design and analysis of WNCSs face unique challenges, a particular one of which is the efficient usage of the shared communication resources[14–16]. As is widely

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known, the more usage of the communication resources usually means the better control system performance, while the not-well-designed usage may harm the other applications that share the communication network. Hence, there exists a trade-off between the communication resource consumption and the control system performance. Consequently, an effective scheduling strategy is needed to achieve the above balance. Furthermore, the well-known data packet dropout issue in conventional NCSs also faces unique challenges for WNCSs in the aforementioned design of the scheduling strategy. Indeed, the transmission success in wireless networks can be usually improved by raising the transmission power which, however, can be either power limited by the battery capacity, or channel limited since the increased power can affect other users.

These challenges have already been investigated considerably to date. On one hand, given the control system setting and usually pre-determined sampling period, one may try to reduce the power consumption by either more from the communication perspective[17–19], or by jointly designing with the controller[20–22]. On the other hand, to reduce data transmissions event-triggered schemes have been the dominant approach[23–27]. Piecewise linear system model, impulsive system model, perturbed linear system model, delay system model, etc. have all been applied jointly with event-triggered schemes to deal with the balance between communication resource consumption and control system performance[28–32].

It is observed that these existing event-triggered control approaches often either fail to take full consideration of the communication constraints, or are lack of the consideration of external disturbances[33–35], while both factors are core to the design and analysis of WNCSs. Motivated by this fact, in this work WNCSs with external disturbances are investigated and a novel predictive event-triggered control (PEC) approach is proposed to balance between the control and communication performances, by digging into further the communication characteristics. This approach combines both a dynamic-dependent, model-based, event-triggered mechanism at the sensor side to reduce data transmissions, and a packet-based controller for better control performance[36]. The trigger threshold is designed especially to keep the difference of state estimation and current state below a proper boundary, and a sufficient condition for ensuring the system uniformly ultimately bounded (UUB) under external disturbance are obtained. The proposed approach is shown to be more robust since external disturbance is explicitly considered.

The remainder of the paper is organized as follows. The problem of interest is formulated and the proposed PEC approach is detailed in Section 2. The stability analysis and the controller gain design method are given in Section 3. Numerical examples illustrate the effectiveness of the proposed approach in Section 4, and the paper is concluded in Section 5.

2 The PEC Approach to WNCSs

This section first presents the problem of interest and then discusses the proposed PEC approach.

2.1 Problem Statement

The considered WNCS is illustrated in Figure 1, where the sensing data sent through the feedback channel (from the sensor to the controller) and the control data sent through the forward channel (from the controller to the actuator), are both transmitted through a shared wireless communication network, i.e., other than the control system of interest, other applications may also have access to the wireless network and hence will compete for the limited wireless communication resources with the considered WNCS. The feedback and forward channels may be part of the same wireless communication networks, but are assumed to be independent from each other since the two channels usually do not compete directly. Also, for wireless networks at a relatively small scale, packet delay can usually be ignored and the main issue is the data packet dropout due to channel collisions. Furthermore, as usually assumed in WNCS no re-transmissions for lost data packets are allowed due to the real-time requirement of control systems.

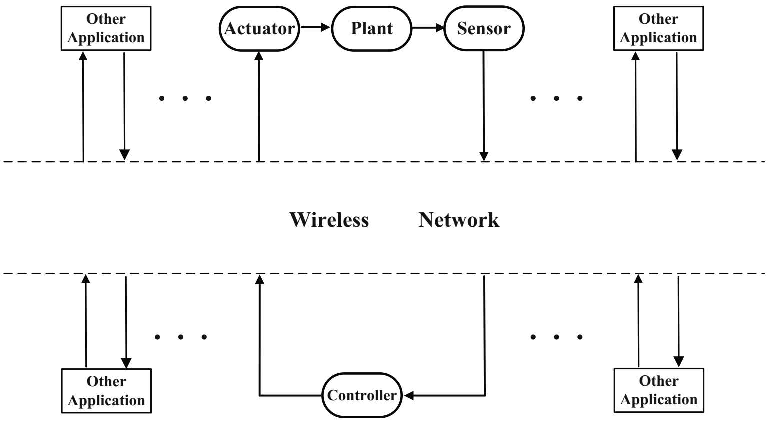


Figure 1 Illustrating wireless networked control systems

Consider the following plant dynamics in discrete-time with disturbance:

$$x(k+1) = Ax(k) + Bu(k) + E\omega(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ are the system state and the control input, respectively. A, B, E are constant matrices with appropriate dimensions, and the disturbance vector $\omega(k) \in \mathbb{R}^q$ is bounded, i.e., $\|\omega(k)\| := \sqrt{\omega(k)^{\mathrm{T}}\omega(k)} \le \omega_{\max}$.

Denote by $d_{sc,k}$ the number of successive sensing data packet dropouts being lost at time k, by $d_{ca,k}$ that of the control data packets, and by $d_k \triangleq d_{ca,k} + d_{sc,k}$ for the round trip. We are safe to assume that d_k is upper bounded by some constant $d_{\max}$, as otherwise the system would be totally open-loop,

$$d_k \le d_{\max}, \quad \forall k. \tag{2}$$

For the WNCS in Figure 1, our objective is to deal with data packet dropout effectively subject to the shared communication resource constraint. This is achieved by the proposed PEC approach as detailed in what follows.

2.2 Design of the PEC Approach

The proposed PEC approach is illustrated in Figure 2. Different from conventional control approaches, the PEC approach specially designs an event trigger to determine whether or not to send the sensing data to reduce consumption of communication resource. Then, by generating and sending a sequence of future control signals to the actuator and then choosing from it the appropriate one, data packet dropout can be actively compensated for.

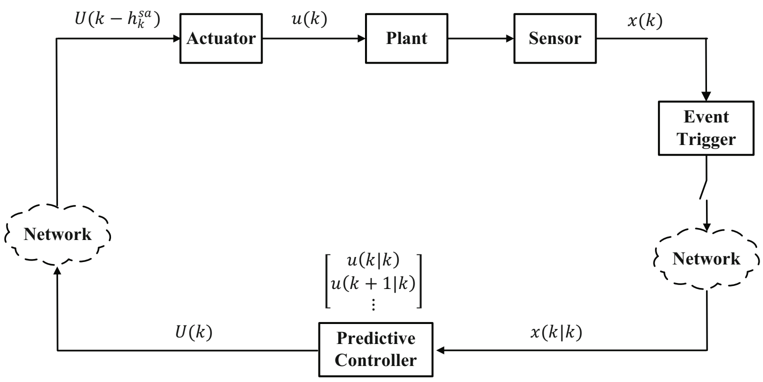


Figure 2 Illustrating the PEC approach to WNCSs

We denote whether the event trigger allows the current system state $x(k)$ to send or not by an indicator $\delta_s(k)$, i.e., $x(k)$ is sent by the event trigger if $\delta_s(k) = 1$, and vice verse. In PEC, $\delta_s(k)$ is determined as follows,

$$\delta_s(k) = \begin{cases} 1, & \|e(k)\| > \lambda \quad \text{or} \quad h_k^s \geq h_{\max}, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $e(k) := \widehat{x}(k) - x(k)$, and $\widehat{x}(k)$, some estimate for $x(k)$, can be generated as follows, with $\widehat{x}(0) = x_0$,

$$\widehat{x}(k+1) = \delta(k)(A + BK)x(k) + (1 - \delta(k))(A + BK)\widehat{x}(k) \tag{4}$$

with $\delta(k)$ being defined later in (12).

It is understood that $e(k)$ should never be too large since otherwise the estimate $\widehat{x}(k)$ fails. The boundedness of $e(k)$ can be guaranteed by defining the following triggering threshold λ (see Lemma 2.1),

$$\lambda = \frac{\sigma\omega_{\max} - \sqrt{d_{\max}}\|\Phi_{d_{\max}}\widetilde{E}\|\omega_{\max}}{\|A^{d_{\max}}\|}, \tag{5}$$

where σ is an adjustable positive number, $\widetilde{E} := \mathrm{diag}(E, E, \cdots, E)$, is a diagonal matrix with appropriate dimension, whose diagonal elements are E, and $\Phi_{d_{\max}} := \left[A^{d_{\max}-1}, \cdots, A, I\right]$. The pre-designed positive number $h_{\max}$ represents the allowed maximum successive time interval of the event trigger being not triggered, and h_k^s is the number of successive time instants i before k with $\delta_s(i) = 0$, i.e.,

$$h_k^s = k - k^s \tag{6}$$

with $k^s \leq k$ being the time instant such that $\delta_s(i) = 0$, $\forall k^s \leq i < k$, $\delta_s(k^s) = 1$, and $\delta_s(0) = 1$. That is, the event trigger is designed mainly using the difference of the estimate and actual system states as the triggering condition, but if the triggering condition has been inactive for more than $h_{\max}$, a forced sensing data transmission will be activated to keep the controller updated.

Define the following indicator $\delta_{sc}(k)$ to indicate whether $x(k)$, if it is allowed to send by the event trigger, is lost due to packet dropout or not, i.e.,

$$\delta_{sc}(k) = \begin{cases} 1, & x(k) \text{ is received by the controller,} \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The predictive controller produces a sequence of forward control predictions, denoted by $U(k)$, and sends it to the actuator, if the latest system state $x(k)$ is available to the controller, and does nothing if otherwise, i.e.,

$$U(k) = \begin{cases} [u(k|k), \cdots, u(k + d_{\max}h_{\max}|k)], & \delta_s(k)\delta_{sc}(k) = 1, \\ \varnothing, & \text{otherwise,} \end{cases} \tag{8}$$

where the forward control signal $u(k + i|k)$ can be designed as follows,

$$u(k + i|k) = Kx(k + i|k) \tag{9}$$

with the prediction of future state $x(k + i|k)$ based on $x(k)$ being

$$x(k + i|k) = \begin{cases} (A + BK)x(k + i - 1|k), & 0 < i \leq d_{\max}h_{\max}, \\ x(k), & i = 0 \;\&\; \delta_s(k)\delta_{sc}(k) = 1. \end{cases} \tag{10}$$

At the actuator side, the actuator follows the packet-based control principle, i.e., it stores only the latest control sequence, and the control signal is carefully chosen from the control sequence to make sure the chosen control signal can exactly compensate for the current delay.

Define the following indicator $\delta_{ca}(k)$ to indicate whether $U(k)$ is received by the actuator due to packet dropout,

$$\delta_{ca}(k) = \begin{cases} 1, & U(k) \text{ is received by the actuator,} \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Define h_k^{sa} as the number of successive time instants i before k with $\delta(i)=0$, i.e.,

$$h_k^{sa}=k-k^{sa} \tag{12}$$

with $k^{sa}<k$ being the time instant such that $\delta(i)=0$, $\forall k^{sa}\le i<k$, $\delta(k^{sa})=1$, and $\delta(k):=\delta_s(k)\delta_{sc}(k)\delta_{ca}(k)$.

By the above design, at time k at the actuator side, the stored control sequence is $U(k-h_k^{sa})$, and hence the actual control signal applied to the plant is

$$u(k)=u(k|k-h_k^{sa})=Kx(k|k-h_k^{sa}). \tag{13}$$

From the assumption $d_k\le d_{\max}$ and pre-designed $h_{\max}$, the actuator can receive at least one control sequence during $d_{\max}h_{\max}+1$ execution times. It is noticed that the packet-based design of $U(k)$ guarantees the availability of the appropriate control signal to the actuator at any give time. It is understood that the exact value of $d_{\max}$ can be difficult to obtain in practice. Fortunately we do not need this exact value to determine the value of U, but any sufficiently large estimate could serve the purpose, despite its conservativeness.

One may notice that the proposed PEC approach can be less conservative compared with conventional event-triggered control approaches, by fitting the event-triggered scheme into the packed-based control framework. Indeed, unlike the use of zero control for most existing event-triggered control in the absence of the current control signal, the PEC approach can take advantage of the packet-based control scheme to have a sequence of predicted control signal always available at the actuator side, thus compensating for the data loss and dealing with external disturbance in an active way.

Lemma 2.1 *For a given positive constant $\sigma>0$ and λ in* (5)*, it holds that*

$$\|e(k)\|\le\sigma\omega_{\max},\quad \forall k. \tag{14}$$

Proof Suppose that $\delta_s(k)=1$, $\delta_{sc}(k)\delta_{ca}=0$, and there are d steps packet lost after k, i.e., $\delta_{sc}(k+1)\delta_{ca}(k+1)=\cdots=\delta_{sc}(k+d)\delta_{ca}(k+d)=0$. According to (10) and (4), $x(k|k-h_k^{sa})$ has the same value as $\widehat{x}(k)$ when $\delta_{sc}(k)\delta_{ca}(k)=0$. Combining this character with the definition of $e(k)$, we have

$$e(k+1)=(1-\delta(k))Ae(k)-E\omega(k), \tag{15}$$

and then

$$e(k+d)=A^d(1-\delta(k))^d e(k)-\Phi_d\widetilde{E}W_d, \tag{16}$$

where $W_d=\Big[\omega(k)^{\mathrm{T}},\omega(k+1)^{\mathrm{T}},\cdots,\omega(k+d-1)^{\mathrm{T}}\Big]^{\mathrm{T}}$. From (16), and $1-\delta(k)\le 1$ we have

$$\|e(k+d)\|\le\|A^d\|\|e(k)\|+\|\Phi_d\widetilde{E}W_d\|. \tag{17}$$

If $\|e(k)\|$ satisfy

$$\|e(k)\| \leq \frac{\sigma\omega_{\max} - \sqrt{d}\|\varPhi_d\widetilde{E}\|\omega_{\max}}{\|A^d\|}, \tag{18}$$

then $\|e(k+d)\| \leq \sigma\omega_{\max}$ holds. Consider the worst case where the event trigger sends $x(k)$ at time k, the control system suffers $d_{\max}$ successive packet dropout after k, it holds that

$$\|e(k)\| \leq \frac{\sigma\omega_{\max} - \sqrt{d_{\max}}\|\varPhi_{d_{\max}}\widetilde{E}\|\omega_{\max}}{\|A^{d_{\max}}\|}, \tag{19}$$

which shows that $\|e(k)\| \leq \sigma\omega_{\max}$ for any given time k, as the trigger threshold λ is chosen as (5). This completes the proof. ∎

3 Stability Analysis and Controller Design

Definition 3.1 (see [37]) The system (1) with the PEC approach is said to be uniformly ultimately bounded (UUB) in a convex and compact set $\mathcal{S}$ which contains the origin in its interior, if there exists $T(x_0)$ for every initial x_0 state such that

$$x(k) \in \mathcal{S}, \quad \forall k \geq T(x_0). \tag{20}$$

3.1 Stability Analysis

The following lemma is first introduced to reveal the relationship between $x(k|k-h_k^{sa})$ and $\widehat{x}(k)$.

Lemma 3.2 *Based on controller design* (9), (10) *and estimation* (4), *it holds that*

$$x(k|k-h_k^{sa}) = \delta(k)x(k) + (1-\delta(k))\widehat{x}(k). \tag{21}$$

Proof According to (10), we have

$$x(k|k-h_k^{sa}) = (A+BK)^{h_k^{sa}}x(k-h_k^{sa}). \tag{22}$$

From (4), if $\delta(k)=0$,

$$\widehat{x}(k) = (A+BK)^{h_k^{sa}}x(k-h_k^{sa}), \tag{23}$$

and if $\delta(k)=1$, then

$$\widehat{x}(k) = \delta(k-1)(A+BK)x(k-1) + (1-\delta(k-1))(A+BK)\widehat{x}(k-1), \tag{24}$$

from (23), then

$$\widehat{x}(k) = \begin{cases} (A+BK)^{h_{k-1}^{sa}+1}x(k-1-h_{k-1}^{sa}), & \delta(k-1)=0 \ \& \ \delta(k)=1, \\ (A+BK)x(k), & \delta(k-1)=1 \ \& \ \delta(k)=1, \\ (A+BK)^{h_k^{sa}}x(k-h_k^{sa}), & \delta(k)=0. \end{cases} \tag{25}$$

Compared (22) with (25), it can be concluded that $\widehat{x}(k)$ and $x(k|k-h_k^{sa})$ have the same value if $\delta(k)=0$, and $x(k|k-h_k^{sa})=x(k)$ if $\delta(k)=1$. This completes the proof. ∎

From Lemma 3.2, the close-loop system can be written as

$$x(k+1)=(A+BK)x(k)+(1-\delta(k))BKe(k)+E\omega(k). \tag{26}$$

Theorem 3.3 *For given positive constants $\sigma\geq 0$, $0<\alpha<1$, and a controller gain K, γ,*

$$\Delta_i=\begin{cases}0, & i=0,\\ 1, & i=1,\end{cases}$$

the system (1) under PEC with trigger threshold (5) is UUB in the set $\xi(P,\gamma^2\omega_{\max}^2)$, where $\xi(P,\gamma^2\omega_{\max}^2):=\{x\in\mathbb{R}^n|x^{\rm T}Px\leq\gamma^2\omega_{\max}^2\}$, if there exist a positive symmetrical matrix $P\in\mathbb{R}^{n\times n}$, positive parameter $\kappa_1\geq 0$, $\kappa_2\geq 0$, $0\leq\kappa_3\leq 1-\alpha$ such that the following matrix inequality is satisfied:

$$\begin{bmatrix}\varXi_{1,1} & * & * & *\\ \Delta_iK^{\rm T}B^{\rm T}PA_{cl} & K^{\rm T}B^{\rm T}PBK-\kappa_1I & * & *\\ E^{\rm T}PA_{cl} & \Delta_iE^{\rm T}PBK & E^{\rm T}PE-\kappa_2I & *\\ 0 & 0 & 0 & \varXi_{4,4}\end{bmatrix}\leq 0, \tag{27}$$

where

$$\varXi_{1,1}=A_{cl}PA_{cl}-(1-\alpha-\kappa_3)P,$$
$$\varXi_{4,4}=\kappa_1\sigma^2\omega_{\max}^2+\kappa_2\omega_{\max}^2-\kappa_3\gamma^2\omega_{\max}^2.$$

Proof Define Lyapunov function candidate $V(k)=x^{\rm T}(k)Px(k)$, and $\triangle V(k)=V(k+1)-V(k)$. For $x(k)\notin\xi(P,\gamma^2\omega_{\max}^2)$, we let $\triangle V(k)\leq-\alpha V(k)$, then,

$$\begin{aligned}\triangle V(k)+\alpha V(k)=&x^{\rm T}(k)\left[A_{cl}^{\rm T}PA_{cl}-(1-\alpha)P\right]x(k)+2x^{\rm T}(k)A_{cl}PE\omega(k)\\ &+2(1-\delta(k))e^{\rm T}(k)K^{\rm T}B^{\rm T}PE\omega(k)+2(1-\delta(k))x^{\rm T}(k)A_{ck}^{\rm T}PBKe(k)\\ &+(1-\delta(k))^2e^{\rm T}(k)K^{\rm T}B^{\rm T}BKe(k)+\omega^{\rm T}(k)E^{\rm T}PE\omega(k)\leq 0,\end{aligned} \tag{28}$$

where $A_{cl}=A+BK$.

Let $\eta(k)=[x^{\rm T}(k)\ e^{\rm T}(k)\ \omega^{\rm T}(k)\ 1]^{\rm T}$, and then (28) can be rewritten as follows:

$$\eta^{\rm T}(k)P_{1,i}\eta(k)\leq 0, \tag{29}$$

where

$$P_{1,i}=\begin{bmatrix}\varXi_{1,1}' & * & * & *\\ \Delta_iK^{\rm T}B^{\rm T}PA_{cl} & K^{\rm T}B^{\rm T}PBK & * & *\\ E^{\rm T}PA_{cl} & \Delta_iE^{\rm T}PBK & \varXi_{3,3}' & *\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{30}$$

with $\Xi'_{1,1} = A_{cl}PA_{cl} - (1-\alpha)P$, $\Xi'_{3,3} = E^{\mathrm{T}}PE$. By Lemma 2.1, (14) is equivalent to

$$\eta^{\mathrm{T}}(k)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma^2\omega_{\max}^2 \end{bmatrix}\eta(k) \le 0. \tag{31}$$

$\|\omega(k)\| \le \omega_{\max}$ can be rewritten as:

$$\eta^{\mathrm{T}}(k)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -\omega_{\max}^2 \end{bmatrix}\eta(k) \le 0. \tag{32}$$

$x(k) \notin \xi(P, \gamma^2\omega_{\max}^2)$ can be written as

$$\eta^{\mathrm{T}}(k)\begin{bmatrix} P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma^2\omega_{\max}^2 \end{bmatrix}\eta(k) \ge 0. \tag{33}$$

By using the S-Procedure, combining (31)–(33) to (29), we have (27) holds.

Now consider $x(k+1) \in \xi(P, \gamma^2\omega_{\max}^2)$ when $x(k) \in \xi(P, \gamma^2\omega_{\max}^2)$. First, $x(k+1)^{\mathrm{T}}Px(k+1) \le \gamma^2\omega_{\max}^2$ can be rewritten by

$$\eta(k)^{\mathrm{T}}P_{2,i}\eta(k) \le 0, \tag{34}$$

where

$$P_{2,i} = \begin{bmatrix} A_{cl}^{\mathrm{T}}PA_{cl} & * & * & * \\ \Delta_i K^{\mathrm{T}}B^{\mathrm{T}}PA_{cl} & K^{\mathrm{T}}B^{\mathrm{T}}PBK & * & * \\ E^{\mathrm{T}}PA_{cl} & \Delta_i E^{\mathrm{T}}PBK & E^{\mathrm{T}}PE & * \\ 0 & 0 & 0 & -\gamma^2\omega_{\max}^2 \end{bmatrix}. \tag{35}$$

$x(k) \in \xi(P, \gamma^2\omega_{\max}^2)$ can be rewritten as

$$\eta^{\mathrm{T}}(k)\begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma^2\omega_{\max}^2 \end{bmatrix}\eta(k) \ge 0. \tag{36}$$

By using the S-Procedure, combining (31), (32), (36) to (34), we have

$$\begin{bmatrix} \Phi_{1,1} & * & * & * \\ \Delta_i K^{\mathrm{T}} B^{\mathrm{T}} P A_{cl} & \Phi_{2,2} & * & * \\ E^{\mathrm{T}} P A_{cl} & \Delta_i E^{\mathrm{T}} P B K & \Phi_{3,3} & * \\ 0 & 0 & 0 & \Phi_{4,4} \end{bmatrix} \leq 0, \tag{37}$$

where $\Phi_{1,1} = A_{cl}^{\mathrm{T}} P A_{cl} - \overline{\kappa}_3$, $\Phi_{2,2} = \Xi_{2,2}$, $\Phi_{3,3} = \Xi_{3,3}$, $\Phi_{4,4} = \kappa_1 \sigma^2 \omega_{\max}^2 + \kappa_2 \omega_{\max}^2 + (\overline{\kappa}_3 - 1)\gamma^2 \omega_{\max}^2$. Note that let $\overline{\kappa}_3 = 1 - \kappa_3$, (27) is sufficient for (37). Thus, (27) also guarantees $x(k+1) \in \xi(P, \gamma^2 \omega_{\max}^2)$ when $x(k) \in \xi(P, \gamma^2 \omega_{\max}^2)$.

When $x(k) \notin \xi(P, \gamma^2 \omega_{\max}^2)$, $V(k)$ satisfies exponential decay as $\triangle V(k) \leq -\alpha V(k)$. Then we can easily obtain that for initial state $x_0 \notin \xi(P, \gamma^2 \omega_{\max}^2)$, $x(k) \in \xi(P, \gamma^2 \omega_{\max}^2)$ for any $k \geq T(x_0)$, and

$$T(x_0) = \frac{\log \gamma^2 \omega_{\max}^2 - \log x_0^{\mathrm{T}} P x_0}{\log(1-\alpha)}. \tag{38}$$

This completes the proof. ∎

Theorem 3.3 gives the UUB conditions for the system (1) under PEC, which is also exponentially stable without disturbance.

3.2 Design of the Controller Gain

Theorem 3.4 *For given constants $\sigma \geq 0$, $0 < \alpha < 1$, and γ,*

$$\Delta_i = \begin{cases} 0, & i = 0, \\ 1, & i = 1, \end{cases}$$

the system (1) *under PEC with threshold function* (5) *is UUB in the set $\xi(P, \gamma^2 \omega_{\max}^2)$, where $\xi(P, \gamma^2 \omega_{\max}^2) := \{x \in \mathbb{R}^n | x^{\mathrm{T}} P x \leq \gamma^2 \omega_{\max}^2\}$, if there exist a positive symmetrical matrix $S \in \mathbb{R}^{n\times n}$, parameters $\kappa_1 \geq 0$, $\kappa_2 \geq 0$, $0 \leq \kappa_3 \leq 1-\alpha$, and matrix $G \in \mathbb{R}^{n\times n}$, $\widetilde{G} \in \mathbb{R}^{m\times n}$ such that*

$$\begin{bmatrix} \Omega_{1,1} & * & * & * & * & * \\ 0 & -G^{\mathrm{T}} - G + \dfrac{1}{\kappa_1} I & * & * & * & * \\ 0 & 0 & -\kappa_2 & * & * & * \\ 0 & 0 & 0 & -\gamma^2 \omega_{\max}^2 + \kappa_2 \omega_{\max}^2 & * & * \\ AG + B\widetilde{G} & \Delta_i B \widetilde{G} & E & 0 & -S & * \\ 0 & 0 & 0 & \sigma & 0 & -\dfrac{1}{\kappa_1 \omega_{\max}^2} \end{bmatrix} \leq 0, \tag{39}$$

where $\Omega_{1,1} = (1-\alpha-\kappa_3)(-G-G^{\mathrm{T}}+S)$. Then Lyapunov matrix $P = S^{-1}$, and the controller gain is designed as $K = \widetilde{G} G^{-1}$.

Proof Consider (27), by using twice Shur-complement, we have (40).

$$\begin{bmatrix} -(1-\alpha-\kappa_3)P & * & * & * & * & * \\ 0 & -\kappa_1 I & * & * & * & * \\ 0 & 0 & -\kappa_2 I & * & * & * \\ 0 & 0 & 0 & -\gamma^2\omega_{\max}^2+\kappa_2\omega_{\max}^2 & * & * \\ A_{cl} & \Delta_i BK & E & 0 & -P^{-1} & * \\ 0 & 0 & 0 & \sigma & 0 & -\dfrac{1}{\kappa_1\omega_{\max}^2} \end{bmatrix} \le 0. \tag{40}$$

Then pre-multiply and after-multiply matrix, $diag(G^{\mathrm{T}}, G^{\mathrm{T}}, I, I, I, I)$ to (40), we have (41).

$$\begin{bmatrix} -(1-\alpha-\kappa_3)G^{\mathrm{T}}PG & * & * & * & * & * \\ 0 & -\kappa_1 G^{\mathrm{T}}G & * & * & * & * \\ 0 & 0 & -\kappa_2 I & * & * & * \\ 0 & 0 & 0 & -\gamma^2\omega_{\max}^2+\kappa_2\omega_{\max}^2 & * & * \\ A_{cl}G & \Delta_i BKG & E & 0 & -P^{-1} & * \\ 0 & 0 & 0 & \sigma & 0 & -\dfrac{1}{\kappa_1\omega_{\max}^2} \end{bmatrix} \le 0. \tag{41}$$

Consider the following inequalities,

$$\kappa(\kappa^{-1}I - G)^{\mathrm{T}}(\kappa^{-1}I - G) \ge 0,$$
$$(P^{-1} - G)^{\mathrm{T}}P(P^{-1} - G) \ge 0,$$

which is equivalent to following inequalities:

$$\kappa G^{\mathrm{T}}G \ge G^{\mathrm{T}} + G - \kappa^{-1}I, \tag{42}$$
$$G^{\mathrm{T}}PG \ge G^{\mathrm{T}} + G - P^{-1}. \tag{43}$$

Take (42), (43) into (41), and let $S := P^{-1}, \widetilde{G} := KG$, then (39) holds. This completes the proof. ▮

One may notice that $\Xi_{1,1}$ in (27) and $\Omega_{1,1}$ in (39) are bilinear. To solve (39), we can try to get solutions by letting κ_3 increase from 0 to $1-\alpha$ with a given step. Based on this idea, the algorithm in [26] can help to get solutions as shown in Algorithm 1. Similarly, (27) can be solved with Algorithm 1 by taking place $t_j = tr(S)$ with $t_j = tr(P)$. More detail discussions about Algorithm 1 can be found in [26].

4 Numerical Examples

In this section, two numerical examples are considered to illustrate the effectiveness of the proposed approach.

Algorithm 1 Finding solution to (39)[26]

Input Total steps N, $i = 0$, $j = 0$, $\kappa_3 = 0$
While $i < N$ **do**
  if (39) has solution **then**
    $t_j = \mathrm{tr}(S)$; % Get trace of matrix S
    $\kappa_{3,j} = \kappa_3$;
    $j = j + 1$;
  endif
  $\kappa_3 = \kappa_3 + \frac{1-\alpha}{N}$;
  $i = i + 1$;
end
$j* = \arg\min_i\{t_i\}$;
output corresponding solution of (39) when $\kappa_3 = \kappa_{3,j*}$.

Case 1 Consider the following discrete model:

$$x(k+1) = Ax(k) + Bu(k) + E\omega(k),$$

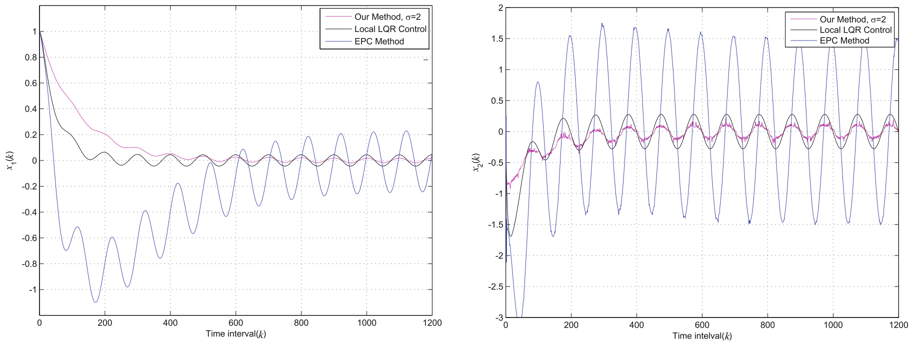
where x_1, x_2 denote the state of pendulum angle and the pendulum angular velocity, respectively, the discrete sampling interval is 0.01 s, and

$$A = \begin{pmatrix} 1.0018 & 0.01 \\ 0.36 & 1.0018 \end{pmatrix}, \quad B = E = \begin{pmatrix} -0.001 \\ -0.184 \end{pmatrix}.$$

The packet transmission in the network may be lost with the probability of 0.2, and the max successive number of packet dropout $d_{\max} = 4$, the maximum transmission interval $h_{\max} = 14$, $\omega(k) = 0.3\sin(0.02\pi k)$. By choosing $\alpha = 0.001$, $\sigma = 2$, and the initial state $x_0 = [1\ 0]^{\mathrm{T}}$, the controller gain is obtained as $K = [7.0600 \quad 3.8924]$.

As Figure 3 shows, our approach behaves better in the presence of disturbance compared with EPC approach[35]. On the other hand, with less data transmissions, our approach still achieves satisfactory performance compared with local LQR method.

For different σ, the triggering rates (steps of triggering/total steps) and the corresponding controller gains are shown in Table 1. Not surprisingly, it shows a trade-off between resource consumption and system performance in the sense that, the greater σ is, the less the data transmissions need, and the poorer the performance will be, as show in Figure 4.



(a) Dynamic comparation of x_1 (b) Dynamic comparation of x_2

Figure 3 System state for our approach, local LQR control, and EPC method

Table 1 The triggering rates and controller gains in 1200 steps simulation under different σ

σ	triggering rate	K
1.5	76.3%	[7.1512 3.8924]
2	48.8%	[7.0600 3.8924]
2.5	40.3%	[6.9719 3.9352]

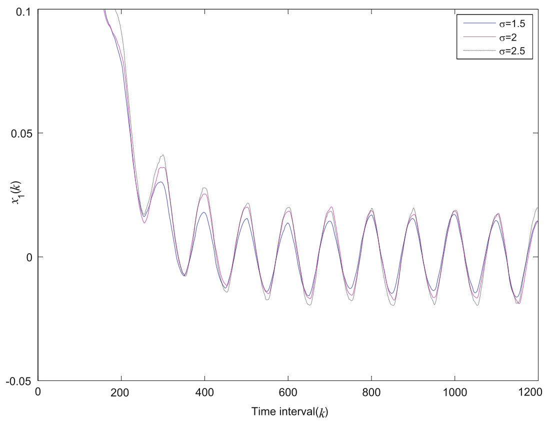


Figure 4 Comparision of with different σ

Case 2 consider the following system matrices:

$$A = \begin{bmatrix} 1.0000 & 0.01 & 0.0001 & 0 \\ 0 & 0.9982 & 0.0267 & 0.0001 \\ 0 & 0 & 1.0016 & 0.01 \\ 0 & -0.0296 & 0.3119 & 1.0016 \end{bmatrix}, \quad B = E = \begin{bmatrix} 0.0001 \\ 0.0182 \\ 0.0002 \\ 0.0454 \end{bmatrix}.$$

The discrete sampling interval is taken as 0.01s, and the network set and disturbance set are the same as the earlier case. Given $\alpha = 0.02$, $\sigma = 0.3$, $\gamma^2 = 6.8$, and the initial state

$x_0 = [0.98\ 0\ 0.2\ 0]^{\mathrm{T}}$, we obtain $K = [18.0855\ 26.8057\ -90.2683\ -17.9167]$. The trigger rate of our approach is 42.7%, and our approach shows more robust behavior compared with the approach in [35], as shown in Figure 5.

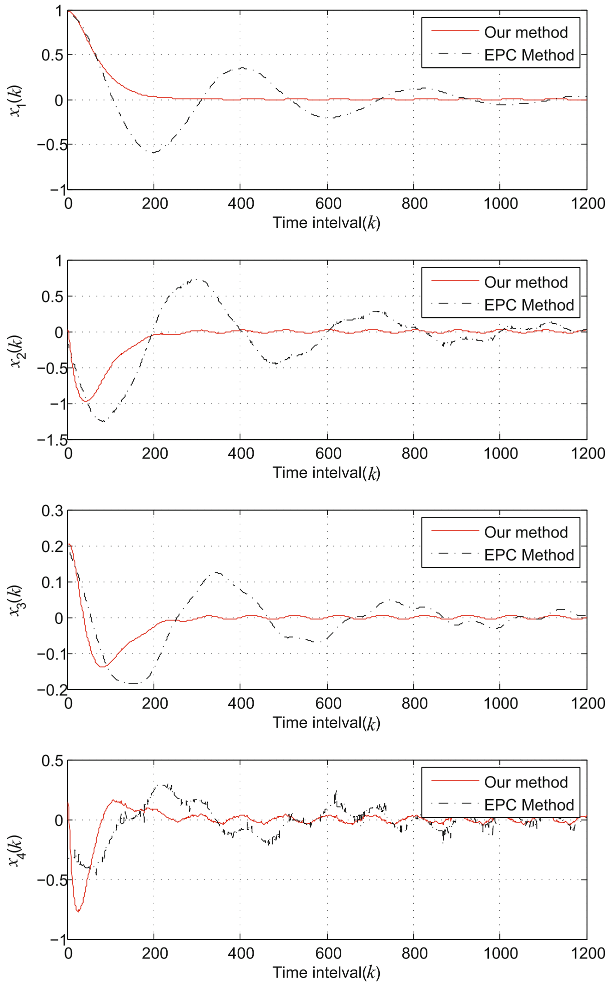


Figure 5 Showing the system dynamics of our approach and the EPC method

5 Conclusions

To reduce the resource consumption while guaranteeing the performance of the control system, we propose a predictive event-triggered control approach for wireless networked control systems subject to packet dropout and disturbance. Such an approach combines a novel event trigger strategy with the packet-based framework. The closed-loop system is both theoretically analyzed and numerically validated.

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